Math 2150 - Homework # 6

Second order linear ODEs - Theory

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1. In this problem we will solve

$$y'' - 7y' + 10y = 24e^x$$

on the interval $I = (-\infty, \infty)$.

(a) Show that $y_1 = e^{2x}$ and $y_2 = e^{5x}$ are linearly independent on I.

(b) Show that $y_1 = e^{2x}$ and $y_2 = e^{5x}$ are solutions to the homogeneous linear equation $y'' - 7y' + 10y = 0$ on I.

(c) Use (a) and (b) above to state the general solution y_h to $y'' - 7y' + 10y = 0$ on I.

(d) Show that $y_p = 6e^x$ is a particular solution to $y'' - 7y' + 10y = 24e^x$ on I.

(e) Give a formula for the general solution to $y'' - 7y' + 10y = 24e^x$ on I.

(f) Find the solution to the initial-value problem

$$y'' - 7y' + 10y = 24e^x, \quad y'(0) = 6,\ y(0) = 0$$

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2. In this problem we will solve

$$x^2y'' - 5xy' + 8y = 24$$

on the interval $I = (-\infty, \infty)$.

(a) Show that $y_1 = x^2$ and $y_2 = x^4$ are linearly independent on I.

(b) Show that $y_1 = x^2$ and $y_2 = x^4$ are solutions to the homogeneous linear equation $x^2y'' - 5xy' + 8y = 0$ on I.

(c) Use (a) and (b) above to state the general solution y_h to $x^2y'' - 5xy' + 8y = 0$ on I.

(d) Show that $y_p = 3$ is a particular solution to $x^2y'' - 5xy' + 8y = 24$ on I.

(e) Give a formula for the general solution to $x^2y'' - 5xy' + 8y = 24$ on I.

(f) Find the solution to the initial-value problem

$$x^2y'' - 5xy' + 8y = 24, \quad y'(1) = 0,\ y(1) = -1$$

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3. In this problem we will solve

$$2x^2y'' + 5xy' + y = x^2 - x$$

on the interval $I = (0, \infty)$.

(a) Show that $y_1 = x^{-1/2}$ and $y_2 = x^{-1}$ are linearly independent on I.

(b) Show that $y_1 = x^{-1/2}$ and $y_2 = x^{-1}$ are solutions to the homogeneous linear equation $2x^2y'' + 5xy' + y = 0$ on I

(c) Use (a) and (b) above to state the general solution y_h to $2x^2y'' + 5xy' + y = 0$ on I.

(d) Show that $y_p = \frac{1}{15}x^2 - \frac{1}{6}x$ is a particular solution to $2x^2y'' + 5xy' + y = x^2 - x$ on I.

(e) Give a formula for the general solution to $2x^2y'' + 5xy' + y = x^2 - x$.

(f) Find the solution to the initial-value problem

$$2x^2y'' + 5xy' + y = x^2 - x, \quad y'(1) = 0,\ y(1) = 0$$
